Relay Subset Selection in Wireless Networks Using Partial Decode-and-Forward Transmission

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Abstract

This paper considers the problem of selecting a subset of nodes in a two-hop wireless network to act as relays in aiding the communication between the source-destination pair. Optimal relay subset selection with the objective of maximizing the overall throughput is a difficult problem that depends on multiple factors including node locations, queue lengths and power consumption. A partial decode-and-forward strategy is applied in this paper to improve the tractability of the relay selection problem and performance of the overall network.

Note that the number of relays selected ultimately determines the performance of the network. This paper benchmarks this performance by determining the net diversity achieved using the relays selected and the partial decode-and-forward strategy. This framework is subsequently used to further transform relay selection into a simpler relay placement problem, and two proximity-based approximation algorithms are developed to determine the appropriate set of relays to be selected in the network. Other selection strategies such as random relay selection and a greedy algorithm that relies on channel state information are also presented. This paper concludes by showing that the proposed proximity-based relay selection strategies yield near-optimal expected rates for a small number of selected relays.

Keywords - Greedy algorithms, partial decode-and-forward, superposition coding, relays.

1 Introduction

Relay-assisted communication is a promising strategy for both centralized and decentralized communication networks [1, 2]. Two-hop relay-based communication is having a considerable influence on emerging standards both in local area networks, IEEE 802.11s [1] and broadband wireless access networks, IEEE 802.16j [2]. Two-hop relay systems consist of a source, a destination and one or more relays where the relay nodes work together as a single set of intermediaries between the source and the destination [3]. Direct transmission occurs between the source and the destination, and the relays assist the source only if the destination cannot decode the direct transmission. There are multiple concrete benefits of introducing these intermediate relays, which include improved system throughput and greater coverage [2]. Multihop relaying [4, 5] is a key enabling technology for networks of the future, but before the performance tradeoffs of multihop relaying can be characterized, it is critical that the issues facing two-hop relaying be fully understood.

Given that the source can enlist multiple nodes to simultaneously act as relays, two questions naturally arise. First, how many relays must the source enlist to aid its transmission to gain the maximum advantage for the resources consumed? Second, which of the nodes in the pre-existing network must be enlisted to act as relays? When multiple-relay selection is allowed, there are numerous tradeoffs that govern system performance [5–7]. While selecting a large number of relays offers the benefit of coherent combining, resulting in increased throughput and thus higher overall quality of service, it suffers from drawbacks as well. Firstly, system resources are drained faster when multiple relays are selected. Second, there are complexity and implementation issues - it is difficult to synchronize the transmissions from multiple disparate relays [8–11], and receiver complexity increases with the number of relays. A single relay can be selected to assist the source transmission [12–19], which offers lower gains in terms of total diversity and rate but is simpler to implement and consumes less power over the entire network. This paper has two goals. One goal is to understand the fundamental limits of multiple-relay selection to benchmark various relay selection algorithms. To this end, we focus on minimizing relay power consumption and treat implementation issues and complexity as a secondary concern.

Regardless of the number of relays selected, it is difficult to determine which node(s) in the network must act as relays to aid the source transmission. For example, selecting the relay with the best channel to the destination may not be an optimal strategy, as this relay may be heavily loaded with traffic and running low on resources. Thus, relay selection is a very difficult problem, as selecting the "optimal" subset from the set of candidate relay nodes is affected by the presence of multiple parameters that govern system performance. In particular, relay node selection often translates to a combinatorial optimization problem [31], which currently does not have an elegant polynomial-time algorithmic solution.

The second goal of this paper is to provide algorithms for relay node selection that serve as a good approximation to the problem of optimal relay selection from the point of view of throughput maximization with power allocation. Moreover, we desire the algorithms to have low complexity and be highly intuitive in terms of design and implementation. Note that any selection algorithm is closely coupled with the transmission strategy employed in the network (such as decode/amplify/compress and forward). Thus, we discuss the transmission strategy employed in this paper and then delve into the details of the algorithms.

In our paper, we use a partial decode-and-forward transmission strategy proposed in [21][1]. Partial decode-and-forward as described in [21] relies on a two-level superposition coding strategy introduced by T. Cover for broadcast channels [26] and further studied in [27–29]. Under this setting, the transmitter employs a layered coding strategy, allowing the receiver to decode the transmitter's message partially if it is incapable of determining it in its entirety. Note that the conventional decode-and-forward strategy as in [22] is a special case of the partial decode-and-forward strategy, and therefore, partial decode-and-forward is a useful tool that has all the properties of decode and forward incorporated into it. In particular, partial decode-and-forward offers both the diversity advantages of amplify-and-forward and the inherent robustness to noise of decode-and-forward [20]. The other main advantage of partial decode-and-forward is the tractability it lends to the relay selection problem.

In this paper, we use the partial decode-and-forward framework as a platform to transform the relay selection problem into a relay-placement problem, whose solution suggests the "best" set of relays to be selected. There are two approximation steps: we first approximate selection of m relays by the problem of finding the m relays that are closest to a rate-maximizing location, and we show that obtaining the rate-maximizing location is equivalent to maximizing a *signomial function* [30]. Since *signomial programs* are, in general, not easy to solve, we further approximate the relay selection problem by the problem of finding the relays that are closest to the rate-maximizing location in a three-node line network. Obtaining this rate-maximizing location is equivalent to maximizing a polynomial over a given range of values, which can be accomplished using deterministic polynomial-time algorithms. This approximation motivates two *proximity-based* algorithms (which we call Multiple Fan Out and Single Fan Out, detailed in Section 4) that

[1]Note that this notion of partial decode-and-forward is distinct from the one in [22] as it is inspired by outage capacity. It is based on the superposition coding strategy for broadcast channels in [25], while the partial decode-and-forward strategy in [22] is derived from block-Markov coding.

select relays based on their proximity to one of the rate-maximizing locations. In addition, we present a greedy selection algorithm (which we call Best Gains, also detailed in Section 4) that chooses relays based on their channel gains to the destination and the amount of the source message that they have decoded. Here, the selected relays must have decoded at least one of the two messages from the source. Note that greedy algorithms are worthy of consideration for the relay selection problem because they can yield the optimal solution for certain relay selection problem settings, including finding a minimum spanning tree in a given graph [31]. Finally, we present a selection algorithm that randomly selects relay nodes (which we call Random Relays, also detailed in Section 4) and compare the performance of all four algorithms - Multiple Fan Out, Single Fan Out, Best Gains and Random Relays.

Once a subset of the candidate relay nodes has been selected to assist the source, we consider the issue of power allocation over the chosen relays. The objective of power allocation over the chosen relays is to minimize the outage probability given that the destination initially cannot decode the entire message from the transmitter. We apply per-node and sum power constraints to the chosen relays. Our focus on outage probability minimization naturally leads us to derive the diversity gain that is achieved by allowing m of the candidate relays to assist the source. The resulting analysis extends the single-relay result in [21] and further highlights the performance benefits of multiple-relay selection. We stress that the derived diversity gain is at most the diversity achieved by selecting m relays out of K_r candidate relays. For example, selecting the relay with the best channel gain to the destination can yield a diversity gain of $K_r + 1$ [16]. We also propose outage probability-minimizing power allocation strategies for the cases where $m = 2$ and $m = 3$ relays are selected to assist the source, and we see that the proximity of each relay to the destination determines the appropriate power allocation strategy that will be employed.

As we are primarily interested in the fundamental limits of relay selection, we allow our transmission strategy to have arbitrarily long codewords that are generated using an i.i.d. Gaussian distribution of suitable variance to meet the overall power constraint. Note that the capacity and thus, the capacity achieving input distribution (if any) of the additive Gaussian noise relay channel is in general unknown. Thus, we choose this coding strategy for two reasons: 1) it has been found to be optimal in most of the special cases whose capacity is known (physically/reversely degraded [22], orthogonal channels [23] and uniform phase fading [24]) and 2) it yields explicit rate expressions and intuitive coding strategies. The destination and all potential relay nodes employ typical set decoding as defined in [32]. All nodes have perfect channel state information and the overhead inherent in our system model can be neglected.

This paper is organized as follows. In Section II we describe the system model and introduce the two-level superposition coding strategy that will be used throughput the paper. In Section III, we present the analytical formulation of the relay selection problem and obtain a closed-form expression for the rate-maximizing relay position in a three-node line network. We present the Multiple Fan Out, Single Fan Out, Best Gains and Random Relays algorithms in Section IV. The diversity analysis and outage probability-minimizing relay power allocation strategies are shown in Section V. We present simulation results in Section VI and conclude the paper in Section VII.

2 System Model

First, we introduce the notation used throughout the paper. $\mathbb{E}$ denotes the mathematical expectation operator. $\ln(\cdot)$ represents the natural logarithm function, $\exp(\cdot)$ represents the exponential function and $\Gamma(\cdot)$ is the gamma function. SNR represents the signal-to-noise ratio. $P(A)$ denotes the probability that an event A occurs. $f^{'}(x)$ denotes the derivative of a function f with respect to its argument x. $\|\mathcal{A}\|$ denotes the cardinality of a set $\mathcal{A}$. $|z|^2$ denotes the absolute square of a complex number z. $f(x) \sim g(x)$ for large values of x represents the fact that $f(x)/g(x) \to 1$ as $x \to \infty$ [20, Pg. 3067].

Consider the two-hop wireless network in Fig. 1. The network consists of a single source t, a single destination r and K_r relays interspersed throughout the region between the source and the destination. Let $d_{i,n}$ denote the distance between nodes i and n. Let $h_{i,n}$ denote the channel between nodes i and n.

2.1 Key Assumptions

We make the following critical assumptions in this paper:

- Each relay operates in a half-duplex mode and employs a single antenna.
- Circularly symmetric complex Gaussian additive noise $n_{i,k}$ with mean 0 and variance σ^2 is present at each receiving node i during time slot k.
- $h_{i,n}$ is a Rayleigh fading channel. Thus, $|h_{i,n}|$ is a Rayleigh-distributed random variable with mean 0 and variance $\mathbb{E}(|h_{i,n}|^2)$. This assumption simplifies our analysis and is typically used in the literature to obtain insights on the performance of real-world wireless systems.
- The source knows the exact channel state for all of the channels in the network in Fig. 1. Each relay knows the exact state of its channel from the source. The destination knows the exact state of its channel from each of the relays and from the source.

- Time is slotted and the channel is constant in every time slot.
- Each time slot is large enough to admit an arbitrarily small probability of error as long as the rate in that state is below the maximum achievable for that state (this is also referred to as the block fading assumption).
- A log-distance path loss model is applied [33]. Let λ_c, d_0, and μ denote the carrier wavelength, the reference distance, and the path loss exponent. Then, the channel gain between nodes i and n is

$$\begin{aligned} \mathbb{E}(|h_{i,n}|^2) &= G_{i,n}^2 \\ &= (\lambda_c/4\pi d_0)^2 (d_{i,n}/d_0)^{-\mu}. \end{aligned} \tag{1}$$

2.2 Partial Decode-and-Forward

All relays perform partial decode-and-forward operations based on the two-level superposition coding strategy in [21]. The source transmits $x_{t,1}$ during the first time slot, where

$$x_{t,1} = x_1 + x_2 \tag{2}$$

and the source allocates power βP_t to x_1 and power $\bar{\beta} P_t$ to x_2, where $\beta \in [0,1]$ and $\bar{\beta} = 1 - \beta$. Note that x_1 and x_2 are codewords from codebooks with elements that are generated i.i.d. according to zero-mean Gaussian distributions with variance βP_t and $\bar{\beta} P_t$, respectively.

The destination and all candidate relay nodes employ typical set decoding to decode x_1 and x_2. The candidate relays and the destination initially attempt to decode x_1. If node i can decode x_1 then it attempts to decode x_2. Two channel thresholds, $|h_1|$ and $|h_2|$, are chosen to determine the set of received rates for this two-level coding strategy. Then, x_1 can be decoded at the rate R_1, where

$$R_1 = \ln\left(1 + \frac{|h_1|^2 \beta P_t}{|h_1|^2 \bar{\beta} P_t + \sigma^2}\right) \tag{3}$$

while x_2 can be decoded at the rate R_2, where

$$R_2 = \ln\left(1 + \frac{|h_2|^2 \bar{\beta} P_t}{\sigma^2}\right). \tag{4}$$

Note that if node i attempts to decode x_1 or x_2 at a higher rate than R_1 or R_2, respectively, the resulting probability of error is bounded away from zero.

The received signals at the candidate relay i and at the destination during time slot 1 are, respectively

$$y_{i,1} = h_{t,i} x_{t,1} + n_{i,1} \tag{5}$$

$$y_{r,1} = h_{t,r} x_{t,1} + n_{r,1}. \tag{6}$$

If the destination can decode both x_1 and x_2, it broadcasts this information to the entire network and the source prepares to send $x_{t,2}$ during time slot 2. If the destination can only decode x_1, or if it cannot decode x_1, it broadcasts this information to the entire network. The source then selects a subset of the candidate relays to assist its transmission.

For relay i, if $|h_{t,i}| < |h_1|$, then it cannot decode x_1 and it does not transmit during time slot 2. If $|h_1| \leq |h_{t,i}| < |h_2|$, then a selected relay i can only decode x_1 and will forward x_1 to the destination during time slot 2. If $|h_{t,i}| \geq |h_2|$, then a selected relay i can decode $x_{t,1}$ and will forward $x_{t,1}$ to the destination. Thus, relay i allocates power P_i to its transmission $x_{r,i}$ to the destination, where

$$x_{r,i} = \begin{cases} 0 & \text{if } |h_{t,i}| < |h_1| \\ \sqrt{\frac{P_i}{\beta_i P_t}} x_1 & \text{if } |h_1| \leq |h_{t,i}| < |h_2| \\ \sqrt{\frac{P_i}{P_t}} (x_1 + x_2) & \text{if } |h_{t,i}| \geq |h_2|. \end{cases} \tag{7}$$

We set $\beta_i = \beta$ for the majority of this paper; in Section 6.1 we investigate the performance impact of varying β_i with respect to β.

Thus, if $\mathcal{A}$ denotes the set of all relays that transmit during time slot 2, the destination receives

$$y_{r,2} = \sum_{i \in \mathcal{A}} h_{i,r} x_{r,i} + n_{r,2} \tag{8}$$

during time slot 2. After time slot 2, if the destination can decode $x_{t,1}$, the received rate is $R_1 + R_2$. If the destination can only decode x_1, the received rate is R_1, and if the destination cannot decode x_1, the received rate is 0. Note that this two-level coding strategy can be generalized to a multiple-level approach based on broadcast strategies introduced for the single user and MAC channels [25]. Once the two-level strategies and algorithms are understood, their generalization to $n > 2$ levels is relatively straightforward but leads to extremely unwieldy expressions. Moreover, it is unclear if using a multiple-level approach will provide significant gains in performance. Thus, we have chosen to limit ourselves to a two-level transmission strategy in this paper.

The objective in this paper is to choose $\mathcal{A}$ to maximize the expected rate subject to a sum power constraint over all relays $i \in \mathcal{A}$. In the next section we motivate two algorithms for selecting $\mathcal{A}$.

3 Rate-Maximizing Relay Position

We formulate the relay selection problem for an arbitrary number of selected relays, and then we show how this problem can be simplified by considering a three-node line network.

3.1 Optimal Relay Placement in General Network

Consider the case where a subset $\mathcal{A}$ of the available relay nodes $\{1, 2, \ldots, K_r\}$ are selected to assist the source. Let h denote the channel between a transmitting node and a receiving node. The received rate at a receiving node via decoding x_1 is [21]

$$C_1(|h|^2) \triangleq \ln\left(1 + \frac{|h|^2\beta P_t}{|h|^2\bar{\beta}P_t + \sigma^2}\right) \tag{9}$$

and the received rate at a receiving node via decoding x_2 after decoding x_1 is [21]

$$C_2(|h|^2) \triangleq \ln\left(1 + \frac{|h|^2\bar{\beta}P_t}{\sigma^2}\right). \tag{10}$$

Let $P_{out}(R_1, \mathcal{A})$ denote the probability that the destination cannot decode x_1 after time slot 2, and let $P_{out}(R_2, \mathcal{A})$ denote the probability that the destination cannot decode x_2 after time slot 2. Then, the expected rate of the two-level superposition coding strategy is [21]

$$\bar{R}_{sc,2}(\mathcal{A}) = (1 - P_{out}(R_1, \mathcal{A}))R_1 + (1 - P_{out}(R_2, \mathcal{A}))R_2 \tag{11}$$

and so the relay selection problem can be formulated as follows

$$\begin{aligned} &\max_{\mathcal{A}\subseteq\{1,2,\ldots,K_r\}} \bar{R}_{sc,2}(\mathcal{A}) \\ &\text{subject to } \sum_{i\in\mathcal{A}} P_i \le P_{max} \text{ and } 0 \le P_i \le P_{i,max} \quad \forall i \in \mathcal{A}. \end{aligned} \tag{12}$$

Let Δ denote the set of all relays that cannot decode x_1, and let Θ denote the set of all relays that can decode x_1 but cannot decode x_2. The probability that the destination cannot decode x_1 after time slot 2 is

$$\begin{aligned} P_{out}(R_1, \mathcal{A}) = &\sum_{(0\le\alpha,\xi\le\|\mathcal{A}\|),\alpha+\xi\le\|\mathcal{A}\|} \left(\sum_{\Delta\subseteq\mathcal{A},\Theta\subseteq\mathcal{A},\|\Delta\|=\alpha,\|\Theta\|=\xi,\Delta\bigcap\Theta=\emptyset} \left(\left(\prod_{\delta\in\Delta} P(C_1(|h_{t,\delta}|^2) < R_1) \right) \right.\right. \\ &\times \left(\prod_{\theta\in\Theta} P(C_1(|h_{t,\theta}|^2) \ge R_1, C_2(|h_{t,\theta}|^2) < R_2) \right) \left.\left(\prod_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))} P(C_2(|h_{t,\eta}|^2) \ge R_2) \right)\right) \\ &\times P\left(\ln\left(1 + \frac{|h_{t,r}|^2\beta P_t + \sum_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))} |h_{\eta,r}|^2\beta P_\eta}{|h_{t,r}|^2\bar{\beta}P_t + \sum_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))} |h_{\eta,r}|^2\bar{\beta}P_\eta + \sigma^2} + \sum_{\theta\in\Theta} \frac{|h_{\theta,r}|^2 P_\theta}{\sigma^2}\right) < R_1 \right)\Bigg) \end{aligned} \tag{13}$$

Each term in the inner sum in (13) represents a scenario where α selected relays cannot decode x_1, ξ selected relays can decode x_1 but cannot decode x_2, and the remaining $\|\mathcal{A}\| - \alpha - \xi$ selected relays can decode x_2.

The expressions in (13) are fairly involved, so we consider the high-SNR regime for ease of analysis. In

Section 5, we prove that

$$P(C_1(|h_{t,\delta}|^2) < R_1)) \sim \frac{1}{G_{t,\delta}^2} \times \frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \tag{14}$$

$$P(C_1(|h_{t,\theta}|^2) \geq R_1, C_2(|h_{t,\theta}|^2) < R_2) \sim 1 \tag{15}$$

$$P(C_2(|h_{t,\eta}|^2) \geq R_2) \sim 1 \tag{16}$$

and

$$P\left(\ln\left(1 + \frac{|h_{t,r}|^2\beta P_t + \sum_{\eta\in(\mathcal{A}\setminus(\Delta\bigcup\Theta))} |h_{\eta,r}|^2\beta P_\eta}{|h_{t,r}|^2\bar{\beta}P_t + \sum_{\eta\in(\mathcal{A}\setminus(\Delta\bigcup\Theta))} |h_{\eta,r}|^2\bar{\beta}P_\eta + \sigma^2} + \sum_{\theta\in\Theta} \frac{|h_{\theta,r}|^2 P_\theta}{\sigma^2}\right) < R_1 \right)$$

$$\sim \left(\frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \right)^{\|\mathcal{A}\|-\alpha+1} \times \frac{1}{(\|\mathcal{A}\| - \alpha + 1)!} \times \frac{1}{G_{t,r}^2} \prod_{\nu\in(\mathcal{A}\setminus\Delta)} \frac{1}{(P_\nu/P_t)G_{\nu,r}^2}. \tag{17}$$

The probability that the destination cannot decode x_2 after time slot 2 is

$$\begin{aligned} P_{out}(R_2, \mathcal{A}) = & \sum_{0\leq\alpha\leq\|\mathcal{A}\|} \left(\sum_{\Delta\subseteq\mathcal{A},\|\Delta\|=\alpha} \left(\left(\prod_{\delta\in\Delta} P(C_2(|h_{t,\delta}|^2) < R_2) \right) \right.\right. \\ & \times \left(\prod_{\theta\in(\mathcal{A}\setminus\Delta)} P(C_2(|h_{t,\theta}|^2) \geq R_2) \right) \\ & \left.\left. \times P\left(C_2\left(|h_{t,r}|^2 + \sum_{\theta\in(\mathcal{A}\setminus\Delta)} |h_{\theta,r}|^2 \right) < R_2 \right) \right) \right). \end{aligned} \tag{18}$$

Each term in the inner sum in (18) represents a scenario where α selected relays cannot decode x_2 and the remaining $\|\mathcal{A}\| - \alpha$ selected relays can decode x_2.

The expressions in (18) are also fairly involved, so we again consider the high-SNR regime for ease of analysis. In Section 5, we prove that

$$P(C_2(|h_{t,\delta}|^2) < R_2) \sim \frac{1}{G_{t,\delta}^2} \times \frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \tag{19}$$

$$P(C_2(|h_{t,\theta}|^2) \geq R_2) \sim 1 \tag{20}$$

and

$$P\left(C_2\left(|h_{t,r}|^2 + \sum_{\theta\in(\mathcal{A}\setminus\Delta)} |h_{\theta,r}|^2 \right) < R_2 \right)$$

$$\sim \left(\frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \right)^{\|\mathcal{A}\|-\alpha+1} \times \frac{1}{(\|\mathcal{A}\| - \alpha + 1)!} \times \frac{1}{G_{t,r}^2} \prod_{\theta\in(\mathcal{A}\setminus\Delta)} \frac{1}{(P_\theta/P_t)G_{\theta,r}^2}. \tag{21}$$

It is apparent that (12) is an optimization problem with linear inequality constraints. Also, from inspecting (13)-(17) it is clear that $P_{out}(R_1, \mathcal{A})$ is a nonlinear function of $P_i\ \forall\ i \in \mathcal{A}$ in the high-SNR regime.

In addition, from inspecting (18)-(21) it is clear that $P_{out}(R_2, \mathcal{A})$ is a nonlinear function of $P_i \ \forall \ i \in \mathcal{A}$ in the high-SNR regime. Then, the preceding analysis shows that in the high-SNR regime, $\bar{R}_{sc,2}(\mathcal{A})$ is a nonlinear function of P_i for $i \in \mathcal{A}$. Thus, nonlinear programming techniques such as sequential quadratic programming [34] can be applied to solve (12) in the high-SNR regime.

The relay selection problem (12) can also be approximated as a relay placement problem where m relays in Fig. 1 are chosen to assist the source. The key idea behind the relay placement problem is to hypothetically place m relays in the locations that would maximize $\bar{R}_{sc,2}(\mathcal{A})$. Then, the m relays in Fig. 1 that are closest to the rate-maximizing locations are selected. It is also assumed that each selected relay i employs the same power $P_i = P_{max}/m$.

To solve for the rate-maximizing locations, recall from (1) that $G_{i,n}^2 = (\lambda_c/4\pi d_0)^2 (d_{i,n}/d_0)^{-\mu} = (d_{i,n})^{-\mu}\chi$. Without loss of generality, assume that the source is located at (0,0) and the destination is located at $(d_{t,r}, 0)$. If relay i is located at (a_i, b_i), then $d_{t,i} = \sqrt{a_i^2 + b_i^2}$ and $d_{i,r} = \sqrt{(d_{t,r} - a_i)^2 + b_i^2}$. In particular, by considering the binomial series $\sum_{k=0}^{\infty}(a+b)^k$ where k is a real number, we see that

$$\begin{aligned} d_{t,i}^{\mu} &= (a_i^2 + b_i^2)^{\mu/2} \\ &= \sum_{k=0}^{\infty} \frac{\Gamma(\mu/2+1)}{k!\Gamma(\mu/2+1-k)} a_i^{2k} b_i^{\mu-2k} \end{aligned} \tag{22}$$

and

$$\begin{aligned} d_{i,r}^{\mu} &= ((d_{t,r} - a_i)^2 + b_i^2)^{\mu/2} \\ &= \sum_{k=0}^{\infty} \frac{\Gamma(\mu/2+1)}{k!\Gamma(\mu/2+1-k)} (d_{t,r} - a_i)^{2k} b_i^{\mu-2k}. \end{aligned} \tag{23}$$

We assume that $0 < a_i < d_{t,r}$ for each relay i since the relays are interspersed throughout the region between the source and the destination. Also, assume without loss of generality that $b_i > 0$ for each relay i since $d_{t,i}$ and $d_{i,r}$ are functions of b_i^2. Let the m selected relays be located at $(a_1, b_1), (a_2, b_2), \ldots, (a_m, b_m)$. Recall from (13), (14), (17), (18), (19) and (21) that in the high-SNR regime, $\bar{R}_{sc,2}(\mathcal{A})$ is a function of $G_{i,n}^{-2} = (d_{i,n})^{\mu}/\chi$. Then, from (22) and (23), we see that $\bar{R}_{sc,2}(\mathcal{A})$ is a function of $\{a_1, b_1, \ldots, a_m, b_m\}$. Since we have assumed that a_i and b_i are positive for each relay i, and the binomial coefficients in (22) and (23) are not necessarily positive, $\bar{R}_{sc,2}(\mathcal{A})$ is a *signomial function* [30] of $\{a_1, b_1, \ldots, a_m, b_m\}$ in the high-SNR regime.

Signomial programs usually do not admit efficient solutions via geometric programming unless the objective function and the associated inequality and equality constraints satisfy certain conditions [30]. Next, we show that given a three-node line network, $\bar{R}_{sc,2}(\mathcal{A})$ reduces to a polynomial function of the relay position d.

3.2 Optimal Relay Placement in Line Network

We consider a line network with $K_r = 1$. The source is located at (0,0), the destination is located at $(d_{t,r}, 0)$ and the relay is located at $(d, 0)$ where $0 < d < d_{t,r}$. The outage probability $P_{out}(R_1, \mathcal{A})$ can be written as [21]

$$\begin{aligned} P_{out}(R_1,\mathcal{A}) &= P(C_1(|h_{t,1}|^2) < R_1)P(C_1(|h_{t,r}|^2) < R_1) \\ &+ P(C_1(|h_{t,1}|^2) \geq R_1, C_2(|h_{t,1}|^2) < R_2) \\ &\quad \times P\left(\ln\left(1 + \frac{|h_{t,r}|^2 \beta P_t}{|h_{t,r}|^2 \bar{\beta} P_t + \sigma^2} + \frac{|h_{1,r}|^2 P_1}{\sigma^2}\right) < R_1 \right) \\ &+ P(C_2(|h_{t,1}|^2) \geq R_2) P\left(C_1\left(|h_{t,r}|^2 + \frac{|h_{1,r}|^2 P_1}{P_t}\right) < R_1 \right) \end{aligned} \tag{24}$$

and the outage probability $P_{out}(R_2, \mathcal{A})$ can be written as [21]

$$\begin{aligned} P_{out}(R_2,\mathcal{A}) &= P(C_2(|h_{t,1}|^2) < R_2)P(C_2(|h_{t,r}|^2) < R_2) \\ &+ P(C_2(|h_{t,1}|^2) \geq R_2) P\left(C_2\left(|h_{t,r}|^2 + \frac{|h_{1,r}|^2 P_1}{P_t}\right) < R_2 \right). \end{aligned} \tag{25}$$

As in Section 3.1, the expressions in (24) and (25) are fairly involved, so we again consider the high-SNR regime for ease of analysis.

In Section 5, we prove that (24) simplifies to

$$\begin{aligned} P_{out}(R_1,\mathcal{A}) \sim &\left(\frac{1}{G_{t,1}^2} \times \frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \right) \left(\frac{1}{G_{t,r}^2} \times \frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \right) \\ + &\left(\frac{1}{(P_1/P_t) G_{1,r}^2} \times \frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \right) \left(\frac{1}{G_{t,r}^2} \times \frac{\exp(R_1) - 1}{(P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1))} \right) \end{aligned} \tag{26}$$

and we prove that (25) simplifies to

$$\begin{aligned} P_{out}(R_2,\mathcal{A}) \sim &\left(\frac{1}{G_{t,1}^2} \times \frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \right) \times \left(\frac{1}{G_{t,r}^2} \times \frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \right) \\ + &\frac{1}{2} \times \left(\frac{1}{(P_1/P_t) G_{1,r}^2} \times \frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \right) \times \left(\frac{1}{G_{t,r}^2} \times \frac{\exp(R_2) - 1}{\bar{\beta}(P_t/\sigma^2)} \right). \end{aligned} \tag{27}$$

Let $G_1 = (\exp(R_1) - 1)/((P_t/\sigma^2) \times (1 - \bar{\beta}\exp(R_1)))$ and $G_2 = (\exp(R_2) - 1)/(\bar{\beta}(P_t/\sigma^2))$. Then

$$\begin{aligned} \bar{R}_{sc,2}(\mathcal{A}) &= (1 - P_{out}(R_1,\mathcal{A}))R_1 + (1 - P_{out}(R_2,\mathcal{A}))R_2 \\ &\sim R_1 \times (1 - G_1^2 \chi^2 \times d_{t,r}^{\mu}(d^{\mu} + (P_t/P_1) \times (d_{t,r} - d)^{\mu})) \\ &+ R_2 \times (1 - G_2^2 \chi^2 \times d_{t,r}^{\mu}(d^{\mu} + (1/2) \times (P_t/P_1) \times (d_{t,r} - d)^{\mu})). \end{aligned} \tag{28}$$

For integral values of the path loss exponent μ, finding the rate-maximizing relay position $\bar{d}$ is equivalent to maximizing a polynomial over $0 < d < d_{t,r}$. For example, if $\mu = 3$, differentiating $\bar{R}_{sc,2}(\mathcal{A})$ with respect to d and setting the result equal to zero yields

$$\begin{aligned} 0 &= \bar{R}'_{sc,2}(\mathcal{A}) && (29)\\ &= -R_1G_1^2\chi^2 d_{t,r}^3(3d^2 - 3(P_t/P_1)(d_{t,r}-d)^2) - R_2G_2^2\chi^2 d_{t,r}^3(3d^2 - (3/2)(P_t/P_1)(d_{t,r}-d)^2) && (30)\\ &= (-3R_1G_1^2\chi^2d_{t,r}^3 + 3R_1G_1^2\chi^2d_{t,r}^3P_t/P_1 - 3R_2G_2^2\chi^2d_{t,r}^3 + 3R_2G_2^2\chi^2d_{t,r}^3P_t/(2P_1))d^2 && (31)\\ &+ (6R_1G_1^2\chi^2d_{t,r}^4P_t/P_1 + 3R_2G_2^2\chi^2d_{t,r}^4P_t/P_1)d + (3R_1G_1^2\chi^2d_{t,r}^5P_t/P_1 + 3R_2G_2^2\chi^2d_{t,r}^5P_t/(2P_1)) \\ &= Ad^2 + Bd + C && (32)\end{aligned}$$

and the solutions of (32) are

$$\bar{d} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \qquad (33)$$

We choose the rate-maximizing relay position to be one of the solutions of (32) that lie in the range $0 < d < d_{t,r}$, assuming that one exists.

4 Relay Subset Selection Algorithms

The analysis in Section 3.1 shows that the combinatorial optimization problem (12) can be approximated by considering a given value of $m \in \{1, 2, \ldots, K_r\}$ and maximizing a signomial function of the relay locations $(a_1, b_1), \ldots, (a_m, b_m)$ in the high-SNR regime, which yields the rate-maximizing set $(\bar{a}_1, \bar{b}_1), \ldots, (\bar{a}_m, \bar{b}_m)$.

Note that since $\bar{R}_{sc,2}(\mathcal{A})$ is a signomial function in the high-SNR regime, relays that are located close to any of the points in the rate-maximizing set should still yield high expected rates due to the inherent smoothness of signomial functions. This motivates the following *proximity-based* algorithm for solving (12).

Algorithm 1. *Multiple Fan Out*

Step 1: For a given value of $m \in \{1, 2, \ldots, K_r\}$, maximize $\bar{R}_{sc,2}(\mathcal{A})$ over all relay locations $(a_1, b_1), \ldots, (a_m, b_m)$ to find the rate-maximizing set $(\bar{a}_1, \bar{b}_1), \ldots, (\bar{a}_m, \bar{b}_m)$.

Step 2: Set $i = 1$ and $\mathcal{A} = \emptyset$.

Step 3: For relay n, where $1 \le n \le K_r$, compute $d(n)$ where $d(n)$ is the distance from relay n to $(\bar{a}_i, \bar{b}_i)$. If relay n is at location (a_n, b_n), $d(n) = \sqrt{(a_n - \bar{a}_i)^2 + (b_n - \bar{b}_i)^2}$.

Step 4: Find the closest relay n to $(\bar{a}_i, \bar{b}_i)$ not in $\mathcal{A}$ and let $\mathcal{A} = \mathcal{A} \cup \{n\}$ and $P_n = P_{max}/m$.

Step 5: If $\|\mathcal{A}\| = m$, stop. Otherwise, let $i = i + 1$ and return to Step 3.

We call the above relay selection algorithm *Multiple Fan Out* because the process of relay selection is analogous to a search party fanning out from its initial location. Here, the objective is to "fan out" from $(\bar{a}_1, \bar{b}_1), \ldots, (\bar{a}_m, \bar{b}_m)$ until m relays have been selected. Note that Step 1 of the *Multiple Fan Out* algorithm involves maximizing a signomial function, which usually does not admit an efficient solution. To obtain a more tractable problem, the analysis in Section 3.2 shows that in the case of a three-node line network, $\bar{R}_{sc,2}(\mathcal{A})$ is a polynomial function of the relay location d in the high-SNR regime. Maximizing $\bar{R}_{sc,2}(\mathcal{A})$ yields the rate-maximizing relay location $\bar{d}$.

Also, since $\bar{R}_{sc,2}(\mathcal{A})$ is a polynomial function in the high-SNR regime for a three-node line network, relays that are located close to the rate-maximizing relay location $\bar{d}$ should still yield high expected rates due to the inherent smoothness of polynomial functions. This motivates another *proximity-based* algorithm for solving (12). Again we assume that the path loss exponent μ takes on an integral value. We also assume that exactly m relays are to be selected, which further simplifies the algorithm.

Algorithm 2. *Single Fan Out*

Step 1: Maximize (28) to find the rate-maximizing relay location $(\bar{d}, 0)$.

Step 2: For relay n, where $1 \le n \le K_r$, compute $d(n)$ where $d(n)$ is the distance from relay n to $(\bar{d}, 0)$. If relay n is at location (a_n, b_n), $d(n) = \sqrt{(a_n - \bar{d})^2 + b_n^2}$.

Step 3: Sort the set of relays $\{1, 2, \ldots, K_r\}$ as $\{a_1, a_2, \ldots, a_{K_r}\}$, where

$$d(a_1) \le d(a_2) \le \cdots \le d(a_{K_r}).$$

Step 4: Find the closest relay n to $(\bar{d}, 0)$ not in $\mathcal{A}$ and let $\mathcal{A} = \mathcal{A} \cup \{n\}$ and $P_n = P_{max}/m$.

Step 5: If $\|\mathcal{A}\| = m$, stop. Otherwise, return to Step 4.

We call the above relay selection algorithm *Single Fan Out* because $\bar{d}$ is computed via analysis of a single-relay line network. Note that both the *Multiple Fan Out* and *Single Fan Out* algorithms are greedy strategies in that the order of procession through the list of relays is based on their proximity to $(\bar{x}_1, \bar{y}_1), \ldots, (\bar{x}_m, \bar{y}_m)$ and $\bar{d}$, respectively. Greedy algorithms are useful for the problem at hand in that they possess an inherent simplicity, and their run-times are usually simple to characterize.

We propose another greedy approach for selecting $\mathcal{A}$. For simplicity, we assume that at most K_{max} relays are to be selected.

Algorithm 3. *Best Gains*

Step 1: For relay i, where $1 \leq i \leq K_r$, compute $|h_{i,r}|^2$.

Step 2: Sort the set of relays $\{1, 2, \ldots, K_r\}$ as $\{a_1, a_2, \ldots, a_{K_r}\}$, where

$$|h_{a_1,r}|^2 \geq |h_{a_2,r}|^2 \geq \cdots \geq |h_{a_{K_r},r}|^2.$$

Step 3: Let $i = 1$ and $\mathcal{A} = \emptyset$.

Step 4: If relay a_i has decoded x_1, then $\mathcal{A} = \mathcal{A} \cup \{a_i\}$ and $P_i = P_{max}/K_{max}$.

Step 5: If $\|\mathcal{A}\| = K_{max}$ or $i = K_r$, stop. Otherwise, let $i = i + 1$ and return to Step 4.

We call the above relay selection algorithm *Best Gains* because the order of procession through the list of relays is based on their channel gains to the destination. The objective is to choose relays that will be able to reliably transmit to the destination during time slot 2. Note that a check is performed on each selected relay in Step 4 to ensure that it will be able to forward at least x_1 to the destination.

To obtain a lower bound on the performance of the above greedy algorithms, we propose the following algorithm whereby relays are randomly selected to transmit during time slot 2.

Algorithm 4. *Random Relays*

Step 1: Let $\mathcal{A} = \emptyset$.

Step 2: Randomly select a relay $i \in \{1, 2, \ldots, K_r\} \setminus \mathcal{A}$ and let $\mathcal{A} = \mathcal{A} \cup \{i\}$ along with $P_i = P_{max}/K_{max}$.

Step 3: If $\|\mathcal{A}\| = K_{max}$, stop. Otherwise, return to Step 2.

Since the destination employs a diversity combining approach to receive the signals from all of the selected relays, the performance of the *Random Relays* algorithm should approach that of the other proposed relay selection algorithms as K_{max} increases.

5 Diversity Performance and Power Allocation Strategies

After m relays are selected to transmit to the destination during time slot 2, we consider the resulting diversity gain and the issue of power allocation over the selected relays. The diversity gain $\kappa(m)$ is obtained by observing that the outage probabilities $P_{out}(R_1, \mathcal{A})$ and $P_{out}(R_2, \mathcal{A})$ are proportional to $SNR^{-\kappa(m)}$ as the SNR P_t/σ^2 approaches infinity. We reiterate that this diversity gain is at most the diversity achieved by selecting m relays out of K_r candidate relays.

The objective of relay power allocation is to minimize the outage probability for decoding x_2 at the destination during time slot 2. As discussed in [21], we focus on the outage probability for x_2 since successful decoding of x_2 by the destination implies that the destination has decoded $x_{t,1}$. For our proposed power allocation strategies, we assume that all of the m selected relays have decoded x_2 so that they can assist the destination in decoding x_2. Thus, given this assumption, the outage probability for decoding x_2 at the destination during time slot 2 is

$$\begin{aligned} P_{\text{out},2}(R_2, m) &= P\Bigg(\ln\left(1 + \frac{|h_{t,r}|^2\bar{\beta}P_t}{\sigma^2} + \sum_{n=1}^{m}\frac{|h_{n,r}|^2\bar{\beta}P_n}{\sigma^2}\right) < R_2\Bigg) && (34)\\ &= P\Bigg(|h_{t,r}|^2 P_t + \sum_{n=1}^{m}|h_{n,r}|^2 P_n < \frac{\sigma^2(e^{R_2}-1)}{\bar{\beta}}\Bigg). && (35)\end{aligned}$$

Let $P_0 = P_t$ and $G_{0,r} = G_{t,r}$. Since we have assumed that $|h_{i,n}|$ is a random variable that is drawn from a Rayleigh distribution, it follows from [35] that

$$P_{\text{out},2}(R_2, m) = \sum_{n=0}^{m}\frac{P_n^m G_{n,r}^{2m}(1-\exp(-\sigma^2(e^{R_2}-1)/(\bar{\beta}P_n G_{n,r}^2)))}{\prod_{k=0,k\neq n}^{m}(P_n G_{n,r}^2 - P_k G_{k,r}^2)}. \tag{36}$$

Thus, the relay power allocation problem is

$$\begin{aligned} &\min_{\{P_n, n=1,2,\ldots,m\}} P_{\text{out},2}(R_2, m) && (37)\\ &\text{subject to} \sum_{n\in\{1,2,\ldots,m\}} P_n \leq P_{max} \text{ and } 0 \leq P_n \leq P_{n,max} \forall n \in \{1,2,\ldots,m\}. \end{aligned}$$

We impose a sum power constraint to minimize the interference with transmissions by neighboring non-network nodes that is caused by transmissions from the selected relays. We study the relay power allocation problem for several values of m.

5.1 Two Relays

Let two relays be selected to transmit during time slot 2. We want to obtain the outage probability-minimizing power allocation strategy for this system. Let $\gamma = \sigma^2(e^{R_2}-1)/\bar{\beta}$. We compute

$$\begin{aligned} P_{\text{out},2}(R_2, 2) = &\frac{P_t^2 G_{t,r}^4\left(1 - e^{-\gamma/(P_t G_{t,r}^2)}\right)}{(P_t G_{t,r}^2 - P_1 G_{1,r}^2)(P_t G_{t,r}^2 - P_2 G_{2,r}^2)} && (38)\\ &+ \frac{P_1^2 G_{1,r}^4\left(1 - e^{-\gamma/(P_1 G_{1,r}^2)}\right)}{(P_1 G_{1,r}^2 - P_t G_{t,r}^2)(P_1 G_{1,r}^2 - P_2 G_{2,r}^2)} + \frac{P_2^2 G_{2,r}^4\left(1 - e^{-\gamma/(P_2 G_{2,r}^2)}\right)}{(P_2 G_{2,r}^2 - P_t G_{t,r}^2)(P_2 G_{2,r}^2 - P_1 G_{1,r}^2)}. \end{aligned}$$

We see that $P_{\text{out},2}(R_2, 2)$ is a fairly complicated function of P_1 and P_2; thus, it is difficult to obtain an exact expression for the $\bar{P}_1$ and $\bar{P}_2$ that minimize $P_{\text{out},2}(R_2, 2)$. Instead, we use numerical methods in the following example to find $\bar{P}_1$ and $\bar{P}_2$ for various network scenarios.

Example 5.1. *Two-Relay Power Allocation*

We place the source at $(0, 0)$ and the destination at $(100, 0)$. We use the Worldwide Interoperability for Microwave Access (WiMAX) signaling bandwidth of 9 MHz [36], and given a noise floor of -204dB/Hz this yields a noise value $\sigma^2 = -134$dB. We also have a carrier frequency f_c = 2.4GHz along with a reference distance d_0 = 1m and a path loss exponent $\mu = 3$. In addition, we set the transmit power to be 98dB above the noise floor of $\sigma^2 = -134$dB, which yields $P_t = 10^{(-134+98)/10}$ and we set the relay sum power constraint $P_{max} = P_t/10$ and $R_2 = 0.1441$.

We divide the line between the source and the destination into three parts. If a relay's position d on the line between the source and the destination is such that $0 \leq d < 100/3$, we say that the relay is in a bad location. Also, if $100/3 \leq d < 200/3$, we say that the relay is in an average location. Finally, if $200/3 \leq d < 100$, we say that the relay is in a good location.

First, consider a case where both relays are in good locations. We place relay 1 at (80,0) and relay 2 at (85,0). Minimizing $P_{\text{out},2}(R_2, 2)$ with respect to P_1 and P_2 while considering the individual and sum power constraints in (37) yields $\bar{P}_1 \approx -49.0114$dB and $\bar{P}_2 \approx -49.0092$dB. In this case, since both relays generally have good channels to the destination, the optimal power allocation is analogous to a transmit diversity strategy.

Next, consider a case where both relays are in bad locations. We place relay 1 at (20,0) and relay 2 at (15,0). Minimizing $P_{\text{out},2}(R_2, 2)$ with respect to P_1 and P_2 while considering the individual and sum power constraints in (37) yields $\bar{P}_1 = P_{max}$ and $\bar{P}_2 = 0$. In this case, since both relays generally have bad channels to the destination, the optimal power allocation is analogous to a multiuser diversity approach. The best strategy is to give all of the available power P_{max} to the relay that generally has a better channel to the destination.

Then, consider a case where one relay is in a good location and the other relay is in a bad location. We place relay 1 at (85,0) and relay 2 at (15,0). Our intuition from the previous example is that relay 1 should receive all of the power P_{max}; indeed, it turns out that $\bar{P}_1 = P_{max}$ and $\bar{P}_2 = 0$.

Next, consider a case where both relays are in average locations. We place relay 1 at (40,0) and relay 2 at (60,0). Minimizing $P_{\text{out},2}(R_2, 2)$ with respect to P_1 and P_2 while considering the individual and sum power constraints in (37) yields $\bar{P}_1 = 0$ and $\bar{P}_2 = P_{max}$. This case demonstrates that if both relays are not in good locations, minimizing the outage probability for decoding x_2 at the destination is analogous to a multiuser diversity approach.

Finally, we fix relay 1 at (95,0) and vary the position of relay 2. Placing relay 2 at (85,0) yields $\bar{P}_1 \approx -49.0085$dB and $\bar{P}_2 \approx -49.0121$dB. Then, moving relay 2 to (70,0) yields $\bar{P}_1 \approx -48.9425$dB and $\bar{P}_2 \approx -49.0791$dB. A steady increase in $\bar{P}_1$ and decrease in $\bar{P}_2$ is observed by moving relay 2 farther away from relay 1 until relay 2 is placed at (42,0), at which point $\bar{P}_2 = 0$. Thus, in this case, the power allocated to relay 1 and relay 2 is a relatively smooth function of the proximity of relay 2 to relay 1.

5.2 General Case

For the diversity analysis in this case, we set the relay powers $P_i = P_t$ for $i \in \{1, 2, \ldots, m\}$ without loss of generality, and so the SNR in this case is P_t/σ^2. Let $\mathcal{A}$ denote the set of m selected relays.

Theorem 1. *Selecting m relays to transmit during time slot 2 yields a diversity gain of $\kappa(m) = m + 1$.*

Proof. See the Appendix. □

As for the outage probability-minimizing power allocation, the intuition that we have gained from the case of $m = 2$ selected relays is that the optimal strategy depends on the locations of the selected relays. In the following example, we let $m = 3$ and demonstrate how P_{max} also plays a role in determining the optimal power allocation.

Example 5.2. *Three-Relay Power Allocation*

We adopt simulation parameters that are similar to those in Example 5.1. Also, as in Example 5.1, we divide the line between the source and the destination into three parts. If a relay's position d on the line between the source and the destination is such that $0 \leq d < 100/3$, we say that the relay is in a bad location. Also, if $100/3 \leq d < 200/3$, we say that the relay is in an average location. Finally, if $200/3 \leq d < 100$, we say that the relay is in a good location.

First, consider a case where all relays are in good locations. We place relay 1 at (80,0), relay 2 at (85,0) and relay 3 at (90,0). Minimizing $P_{\text{out},2}(R_2)$ with respect to P_1, P_2 and P_3 while considering the individual and sum power constraints in (37) yields $\bar{P}_1 \approx -50.8396$dB, $\bar{P}_2 \approx -50.8396$dB and $\bar{P}_3 \approx -50.6376$dB. Again, since all relays generally have good channels to the destination, the optimal power allocation is analogous to a transmit diversity strategy.

Next, consider a case where all relays are in bad locations. We place relay 1 at (10,0), relay 2 at (15,0) and relay 3 at (20,0). Minimizing $P_{\text{out},2}(R_2)$ with respect to P_1 and P_2 while considering the individual and sum power constraints in (37) yields $\bar{P}_1 = 0$, $\bar{P}_2 = 0$ and $\bar{P}_3 = P_{max}$. Again, since all relays generally have bad channels to the destination, the optimal power allocation is analogous to a multiuser diversity approach. The best strategy is to give all of the available power P_{max} to the relay that generally has the best channel to the destination.

Then, consider a case where one relay is in a good location, another relay is in an average location and the third relay is in a bad location. We place relay 1 at (90,0), relay 2 at (60,0) and relay 3 at (30,0). Our intuition from the previous example is that relay 1 should receive all of the power P_{max}; indeed, it turns out that $\bar{P}_1 = P_{max}$, $\bar{P}_2 = 0$ and $\bar{P}_3 = 0$.

We further investigate the case where each relay has a different "location quality". Now, we set the transmit power to be 102dB above the noise floor of $\sigma^2 = -134$dB, which yields $P_t = 10^{(-134+102)/10}$. We also have $R_2 = 0.3954$. If the relay sum power constraint $P_{max} = P_t/10$, we find that $\bar{P}_1 = -42.0663$dB, $\bar{P}_2 = -60.2045$dB and $\bar{P}_3 = 0$.

Also, if we set the transmit power to be 104dB above the noise floor of $\sigma^2 = -134$dB, we have $P_t = 10^{(-134+104)/10}$. By setting $R_2 = 0.5764$ and the relay sum power constraint $P_{max} = P_t/10$, we find that $\bar{P}_1 = -40.0634$dB, $\bar{P}_2 = -58.4013$dB and $\bar{P}_3 = 0$. Thus, as the relay sum power constraint increases, the optimal strategy begins to deviate from the multiuser diversity approach.

6 Simulation Results

6.1 Performance of Relay Selection Algorithms

We employ similar system parameters as in Example 5.1. All transmitting nodes use a carrier frequency $f_c = 2.4$GHz. The reference distance is $d_0 = 1$m, the separation between the source and the destination is $d_{t,r} = 100$m and the path loss exponent $\mu = 3$. We randomly place $K_r = 20$ relays in the region between the source and the destination. All transmissions use the WiMAX signaling bandwidth of roughly 9 MHz.

Fig. 2 shows how the expected rate $\bar{R}_{sc,2}(\mathcal{A})$ varies with the number of selected relays $\|\mathcal{A}\| = m$ for the algorithms that we have proposed. Here we fix the source's power $P_t = -34$dB. The fraction of the source's power allocated to x_1 is $\beta = 0.75$ and the decoding thresholds for x_1 and x_2 are $|h_1| = -101.2378$dB and $|h_2| = -99.0309$dB, respectively.

We see that the greedy *Best Gains* algorithm yields the highest expected rate for all of the proposed selection strategies. This is due to the fact that the *Best Gains* algorithm biases relay selection towards those relays that have good channel gains to the destination and can also transmit during time slot 2; this minimizes the chances of an outage event occurring at the destination where it cannot decode x_1. Also, the *Single Fan Out* algorithm offers virtually the same performance as the *Multiple Fan Out* algorithm, which demonstrates the utility of our simplifications of the relay selection problem. Here, the relays that are close to the rate-maximizing set $(\bar{a}_1, \bar{b}_1), \ldots, (\bar{a}_m, \bar{b}_m)$ for the *Multiple Fan Out* algorithm are also close to the rate-maximizing position $(\bar{d}, 0)$ for the *Single Fan Out* algorithm. In addition, as the number of selected relays $\|\mathcal{A}\|$ increases, each strategy yields a higher expected rate which approaches the maximum expected rate. Finally, note that the performance gap between all of the proposed strategies decreases as the number of selected relays increases. This is due to the fact that selecting multiple relays yields an SNR gain at the destination that gradually overcomes the loss from selecting relays that might not be close to the rate-maximizing positions that are computed by the *Multiple Fan Out* and *Single Fan Out* algorithms.

Fig. 3 shows how the expected rate $\bar{R}_{sc,2}(\mathcal{A})$ varies with the number of selected relays $\|\mathcal{A}\| = m$ for two relay power allocation strategies. We use the same system parameters as in Fig. 2, except that we randomly place m relays in the region between the source and the destination instead of $K_r = 20$ relays. The Optimal Power Allocation strategy entails solving the relay selection problem in (12), and the Equal Power Allocation strategy assigns equal power to all of the selected relays. We also set $P_{max} = P_t$.

We observe that the Equal Power Allocation strategy offers comparable performance to the Optimal Power Allocation strategy. This illustrates the utility of low-complexity strategies that reduce the computation time inherent to interior-point methods that are needed to solve the optimization problem (12).

Fig. 4 shows how the expected rate $\bar{R}_{sc,2}(\mathcal{A})$ varies with the average received SNR at the destination for different ratios between the relays' and source's powers. When the average received SNR values at the destination are 0dB, 2dB and 4dB, the source's power takes on values $P_t = -36$dB, $P_t = -34$dB and $P_t = -32$dB, respectively.

We see that as the average received SNR at the destination increases, the expected rate increases for each value of P_i/P_t. Note that for a fixed value of P_i/P_t, increasing the average received SNR at the destination entails increasing P_i and P_t. For a fixed value of the average received SNR at the destination, the expected rate decreases as P_i/P_t decreases, which corresponds to a decrease in P_i. Thus, even though the three selected relays yield an SNR gain at the destination in time slot 2, this gain decreases as the relays' power decreases.

Fig. 5 shows how the expected rate $\bar{R}_{sc,2}(\mathcal{A})$ varies with the average received SNR at the destination for different values of the relays' power split β_i. We set $\beta = 0.75$.

We see that as the relays' power split β_i decreases, the expected rate increases for all average received SNR values at the destination. Note that as β_i decreases, $\bar{\beta}_i$ increases, which leads to an increase in R_2 as seen in (4). On the other hand, as $\bar{\beta}_i$ increases, (3) shows that R_1 decreases. Fig. 5 shows that the increase in R_2 overcomes the decrease in R_1.

6.2 Performance of Power Allocation Strategies

We employ similar simulation parameters as in Section 6.1, except that here we do not randomly place relays in the region between the source and the destination. We set $\beta = 0.75$ and the relay sum power constraint $P_{max} = P_t/10$. Here, we consider the outage probability for decoding x_2 at the destination, which we define as the probability that the destination does not decode x_2, given that all of the selected relays have decoded x_2. The purpose of this definition is to compare the performance of the power allocation strategies from Section 5.

We adopt the terminology of good, average, and bad relay locations from Example 5.1. First, we place one relay at a good location (85,0) and another relay at a bad location (15,0). Fig. 6 shows how the outage probability varies with the average received SNR at the destination for two power allocation algorithms. The optimal power allocation strategy is chosen to minimize $P_{\text{out},2}(R_2, 2)$, while the equal power allocation strategy gives power $P_{max}/2$ to each relay.

We see that the optimal power allocation strategy has an outage performance that is slightly better than that of the equal power allocation strategy. In this case, the equal power allocation approach performs fairly well because the relay at (85,0) generally has a good channel to the destination; thus, even if this relay only receives power $P_{max}/2$, the destination will still be significantly aided in terms of decoding x_2.

Note that $P_{\text{out},2}(R_2, 2)$ does not monotonically decrease with the average received SNR at the destination. From inspecting the event $P(X < x)$ in (34), it is apparent that X monotonically increases with the average received SNR if all other parameters in X are held constant. Referring to (4), though, shows that x also monotonically increases with the average received SNR. Thus, $P_{\text{out},2}(R_2, 2)$ is not necessarily a decreasing function of the average received SNR.

Next, we place two relays at (80,0) and (85,0). Fig. 7 shows how the outage probability varies with the average received SNR at the destination. Again, the optimal power allocation strategy is chosen to minimize $P_{\text{out},2}(R_2, 2)$, while the single-relay allocation gives power P_{max} to the relay at (85,0).

In this case, the optimal power allocation approach significantly outperforms the single-relay power allocation approach. Since the optimal approach is analogous to a transmit diversity strategy, we note that the low likelihood of both relay-to-destination channels being bad allows the optimal approach to reap significant gains over the naive single-relay approach.

Again, note that $P_{\text{out},2}(R_2, 2)$ does not monotonically decrease with the average received SNR at the destination. This can be similarly explained as in Fig. 6.

Finally, we consider the case of three selected relays. We place one relay at (90,0), another relay at (60,0) and the third relay at (30,0). Fig. 8 shows how the outage probability varies with the average received SNR at the destination for the optimal and equal power allocation strategies.

We see that the optimal strategy has an outage performance that is slightly better than that of the equal power allocation strategy. Again, the equal power allocation approach performs fairly well because the relay at (90,0) generally has a good channel to the destination; thus, even if this relay only receives power $P_{max}/3$, it will still assist the destination in terms of decoding x_2. In addition, the relay at (60,0) will generally be able to assist the destination in terms of decoding x_2, albeit to a lesser degree than the relay at (90,0).

7 Conclusion

We have studied the problem of selecting a set of relay nodes to forward data in a two-hop wireless network. We have considered a scenario where all relay nodes perform partial decode-and-forward operations based on a superposition coding strategy. For this setup, we have shown that relay selection can be initially approximated by the problem of finding the relays that are close to a rate-maximizing location. Finding the rate-maximizing location is usually computationally intensive, so we further simplify the relay selection problem by solving for the rate-maximizing location in a three-node line network. These results motivate two *proximity-based* relay selection algorithms, where relays are chosen to forward data based on their proximity to one of the rate-maximizing locations. We also demonstrated that the *proximity-based* algorithms outperform other selection algorithms such as random relay selection and a greedy strategy that is based on channel state information. In addition, we derived the diversity gain achieved by having multiple relays assist the source and obtained outage probability-minimizing power allocation strategies for the selected relays that are based on the proximity of each relay to the destination. We also illustrated the performance impact of varying system parameters such as the ratio between the relays' and source's powers.

As noted in the Introduction, selecting the optimal subset of candidate relay nodes to assist a source is a difficult problem, and the proposed selection strategies are mainly intended to offer key insights. For example, the expression for the rate-maximizing relay position for the three-node line network motivates intelligent relay placement in a general two-hop network. System designers can reap gains in terms of the average rate by placing relays in close proximity to the rate-maximizing position such that a minimal level of interference between relay transmissions is achieved. Also, the information-theoretic analysis in this paper can be modified to support more practical transmission strategies. By applying cutting-edge coding strategies such as punctured low-density parity-check (LDPC) and turbo codes, the superposition coding approach that is employed in this paper can form the basis of a hybrid-ARQ strategy in a multihop network.

A Proof of Theorem 1

The probability that the destination cannot decode x_1 after time slot 2 is

$$\begin{aligned}P_{out}(R_1,\mathcal{A}) &= \sum_{(0\le\alpha,\xi\le m),\alpha+\xi\le m}\left(\sum_{\Delta\subseteq\mathcal{A},\Theta\subseteq\mathcal{A},\|\Delta\|=\alpha,\|\Theta\|=\xi,\Delta\bigcap\Theta=\emptyset}\left(\prod_{\delta\in\Delta}P(C_1(|h_{t,\delta}|^2)<R_1)\right)\right.\\&\times\left(\prod_{\theta\in\Theta}P(C_1(|h_{t,\theta}|^2)\ge R_1,C_2(|h_{t,\theta}|^2)<R_2)\right)\left(\prod_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))}P(C_2(|h_{t,\eta}|^2)\ge R_2)\right)\\&\left.\times P\left(\ln\left(1+\frac{|h_{t,r}|^2\beta P_t+\sum_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))}|h_{\eta,r}|^2\beta P_t}{|h_{t,r}|^2\bar{\beta}P_t+\sum_{\eta\in(\mathcal{A}\backslash(\Delta\bigcup\Theta))}|h_{\eta,r}|^2\bar{\beta}P_t+\sigma^2}+\sum_{\theta\in\Theta}\frac{|h_{\theta,r}|^2P_t}{\sigma^2}\right)<R_1\right)\right).\end{aligned} \quad (39)$$

Each term in the inner sum in (39) represents a scenario where α selected relays cannot decode x_1, ξ selected relays can decode x_1 but cannot decode x_2, and the remaining $m-\alpha-\xi$ selected relays can decode x_2.

Note that for a Rayleigh fading channel h,

$$\begin{aligned}P(C_1(|h|^2)<R_1) &= P\left(\ln\left(1+\frac{|h|^2\beta P_t}{|h|^2\bar{\beta}P_t+\sigma^2}\right)<R_1\right)\\&= P\left(|h|^2<\frac{\exp(R_1)-1}{1-\bar{\beta}\exp(R_1)}\times\frac{\sigma^2}{P_t}\right)\\&\sim \frac{1}{\mathbb{E}(|h|^2)}\times\frac{\exp(R_1)-1}{(1-\bar{\beta}\exp(R_1))P_t/\sigma^2}\end{aligned} \quad (40)$$

where (40) follows from [20, Fact 1].

Also, for a Rayleigh fading channel h,

$$\begin{aligned}P(C_1(|h|^2)\ge R_1,C_2(|h|^2)<R_2) &\le P(C_1(|h|^2)\ge R_1)\\&\sim 1.\end{aligned} \quad (41)$$

In addition, for independent Rayleigh fading channels h_1 and h_2, note that

$$\begin{aligned} P\left(\ln\left(1+\frac{|h_1|^2\beta P_t}{|h_1|^2\bar{\beta}P_t+\sigma^2}+\frac{|h_2|^2P_t}{\sigma^2}\right)<R_1\right) &\le P\left(\ln\left(1+\frac{|h_1|^2\beta P_t+|h_2|^2\beta P_t}{|h_1|^2\bar{\beta}P_t+|h_2|^2\bar{\beta}P_t+\sigma^2}\right)<R_1\right) \\ &= P(C_1(|h_1|^2+|h_2|^2)<R_1) \\ &= P\left(|h_1|^2+|h_2|^2<\frac{\exp(R_1)-1}{1-\bar{\beta}\exp(R_1)}\times\frac{\sigma^2}{P_t}\right) \\ &\sim \frac{1}{2\mathbb{E}(|h_1|^2)\mathbb{E}(|h_2|^2)}\left(\frac{\exp(R_1)-1}{(1-\bar{\beta}\exp(R_1))P_t/\sigma^2}\right)^2 \end{aligned} \tag{42}$$

where (42) follows from [20, Fact 2].

Also, for a Rayleigh fading channel h,

$$P(C_2(|h|^2)\ge R_2)\sim 1. \tag{43}$$

In addition, for independent Rayleigh fading channels h_1, h_2 and h_3, note that

$$\begin{aligned} P\left(\ln\left(1+\frac{|h_1|^2\beta P_t}{|h_1|^2\bar{\beta}P_t+\sigma^2}+\frac{|h_2|^2P_t}{\sigma^2}+\frac{|h_3|^2P_t}{\sigma^2}\right)<R_1\right) &\le P\left(\ln\left(1+\frac{|h_1|^2\beta P_t+|h_2|^2\beta P_t+|h_3|^2\beta P_t}{|h_1|^2\bar{\beta}P_t+|h_2|^2\bar{\beta}P_t+|h_3|^2\bar{\beta}P_t+\sigma^2}\right)<R_1\right) \\ &= P(C_1(|h_1|^2+|h_2|^2+|h_3|^2)<R_1) \\ &= P\left(|h_1|^2+|h_2|^2+|h_3|^2<\frac{\exp(R_1)-1}{1-\beta\exp(R_1)}\times\frac{\sigma^2}{P_t}\right) \\ &\sim \frac{1}{6\mathbb{E}(|h_1|^2)\mathbb{E}(|h_2|^2)\mathbb{E}(|h_3|^2)}\left(\frac{\exp(R_1)-1}{(1-\bar{\beta}\exp(R_1))P_t/\sigma^2}\right)^3 \end{aligned} \tag{44}$$

where (44) follows from [6, Appendix B].

We use (40), (41), (42), (43) and (44) to see that

$$P\left(\ln\left(1+\frac{|h_{t,r}|^2\beta P_t+\sum_{\eta\in(\mathcal{A}\setminus(\Delta\bigcup\Theta))}|h_{\eta,r}|^2\beta P_t}{|h_{t,r}|^2\bar{\beta}P_t+\sum_{\eta\in(\mathcal{A}\setminus(\Delta\bigcup\Theta))}|h_{\eta,r}|^2\bar{\beta}P_t+\sigma^2}+\sum_{\theta\in\Theta}\frac{|h_{\theta,r}|^2P_t}{\sigma^2}\right)<R_1\right) \tag{45}$$

$$\begin{aligned} &\le P\left(\ln\left(1+\frac{|h_{t,r}|^2\beta P_t+\sum_{\nu\in(\mathcal{A}\setminus\Delta)}|h_{\nu,r}|^2\beta P_t}{|h_{t,r}|^2\bar{\beta}P_t+\sum_{\nu\in(\mathcal{A}\setminus\Delta)}|h_{\nu,r}|^2\bar{\beta}P_t+\sigma^2}\right)<R_1\right) \\ &= P\left(C_1\left(|h_{t,r}|^2+\sum_{\nu\in(\mathcal{A}\setminus\Delta)}|h_{\nu,r}|^2\right)<R_1\right) \\ &\sim \frac{1}{(m-\alpha+1)!}\times\frac{1}{\mathbb{E}(|h_{t,r}|^2)}\times\left(\frac{\exp(R_1)-1}{(1-\bar{\beta}\exp(R_1))P_t/\sigma^2}\right)^{-(m-\alpha+1)}\prod_{\nu\in(\mathcal{A}\setminus\Delta)}\frac{1}{\mathbb{E}(|h_{\nu,r}|^2)}. \end{aligned}$$

Thus, the high-SNR behavior of $P_{out}(R_1,\mathcal{A})$ is

$$\begin{aligned} P_{out}(R_1,\mathcal{A}) &\sim \left(\left(\frac{P_t}{\sigma^2}\right)^{-1}\right)^{\alpha}\times(1)^{\beta}\times(1)^{m-\alpha-\beta}\times\left(\frac{P_t}{\sigma^2}\right)^{-(m-\alpha+1)} \\ &= \left(\frac{P_t}{\sigma^2}\right)^{-(m+1)} \end{aligned} \tag{46}$$

and so we obtain a diversity gain of $\kappa(m) = m + 1$ for decoding x_1 at the destination.

The probability that the destination cannot decode x_2 after time slot 2 is

$$\begin{aligned} P_{out}(R_2, \mathcal{A}) &= \sum_{0\le\alpha\le m} \left(\sum_{\|\Delta\|=\alpha, \Delta\subseteq\mathcal{A}} \left(\prod_{\delta\in\Delta} P(C_2(|h_{t,\delta}|^2) < R_2) \right) \right. \\ &\quad \times \left(\prod_{\theta\in(\mathcal{A}\backslash\Delta)} P(C_2(|h_{t,\theta}|^2) \ge R_2) \right) \times \\ &\quad \left. \times P\left(C_2\left(|h_{t,r}|^2 + \sum_{\theta\in(\mathcal{A}\backslash\Delta)} |h_{\theta,r}|^2 \right) < R_2 \right) \right). \end{aligned} \tag{47}$$

Each term in the inner sum in (47) represents a decoding scenario where α selected relays cannot decode x_2 and the remaining $m - \alpha$ selected relays can decode x_2.

Note that for a Rayleigh fading channel h,

$$\begin{aligned} P(C_2(|h|^2) < R_2) &= P\left(\ln\left(1 + \frac{|h|^2 \bar{\beta} P_t}{\sigma^2} \right) < R_2 \right) \\ &= P\left(|h|^2 < \frac{\exp(R_2) - 1}{\bar{\beta}} \times \frac{\sigma^2}{P_t} \right) \\ &\sim \frac{1}{\mathbb{E}(|h|^2)} \times \frac{\exp(R_2) - 1}{\bar{\beta} P_t / \sigma^2} \end{aligned} \tag{48}$$

where (48) follows from [20, Fact 1].

Also, for independent Rayleigh fading channels h_1 and h_2, note that

$$\begin{aligned} P(C_2(|h_1|^2 + |h_2|^2) < R_2) &= P\left(\ln\left(1 + \frac{|h_1|^2 \bar{\beta} P_t}{\sigma^2} + \frac{|h_2|^2 \bar{\beta} P_t}{\sigma^2} \right) < R_2 \right) \\ &= P\left(|h_1|^2 + |h_2|^2 < \frac{\exp(R_2) - 1}{\bar{\beta}} \times \frac{\sigma^2}{P_t} \right) \\ &\sim \frac{1}{2\mathbb{E}(|h_1|^2)\mathbb{E}(|h_2|^2)} \left(\frac{\exp(R_2) - 1}{\bar{\beta} P_t / \sigma^2} \right)^2 \end{aligned} \tag{49}$$

where (49) follows from [20, Fact 2].

In addition, for independent Rayleigh fading channels h_1, h_2 and h_3, note that

$$\begin{aligned} P(C_2(|h_1|^2 + |h_2|^2 + |h_3|^2) < R_2) &= P\left(\ln\left(1 + \frac{|h_1|^2 \bar{\beta} P_t}{\sigma^2} + \frac{|h_2|^2 \bar{\beta} P_t}{\sigma^2} + \frac{|h_3|^2 \bar{\beta} P_t}{\sigma^2} \right) < R_2 \right) \\ &= P\left(|h_1|^2 + |h_2|^2 + |h_3|^2 < \frac{\exp(R_2) - 1}{\bar{\beta}} \times \frac{\sigma^2}{P_t} \right) \\ &\sim \frac{1}{6\mathbb{E}(|h_1|^2)\mathbb{E}(|h_2|^2)\mathbb{E}(|h_3|^2)} \left(\frac{\exp(R_2) - 1}{\bar{\beta} P_t / \sigma^2} \right)^3 \end{aligned} \tag{50}$$

where (50) follows from [6, Appendix B].

We use (43), (48), (49) and (50) to see that

$$P\left(C_2\left(|h_{t,r}|^2+\sum_{\theta\in(\mathcal{A}\setminus\Delta)}|h_{\theta,r}|^2\right)<R_2\right) \tag{51}$$

$$\sim \frac{1}{(m-\alpha+1)!}\times\frac{1}{\mathbb{E}(|h_{t,r}|^2)}\times\left(\frac{\exp(R_2)-1}{\bar{\beta}P_t/\sigma^2}\right)^{-(m-\alpha+1)}\prod_{\nu\in(\mathcal{A}\setminus\Delta)}\frac{1}{\mathbb{E}(|h_{\nu,r}|^2)}.$$

Thus, the high-SNR behavior of $P_{out}(R_2,\mathcal{A})$ is

$$\begin{aligned} P_{out}(R_2,\mathcal{A}) &\sim \left(\left(\frac{P_t}{\sigma^2}\right)^{-1}\right)^{\alpha}\times(1)^{m-\alpha}\times\left(\frac{P_t}{\sigma^2}\right)^{-(m-\alpha+1)} \\ &= \left(\frac{P_t}{\sigma^2}\right)^{-(m+1)} \end{aligned} \tag{52}$$

and so we obtain a diversity gain of $\kappa(m)=m+1$ for decoding x_2 at the destination.

Thus, we conclude that selecting m relays allows us to reap an overall diversity gain of $\kappa(m)=m+1$.

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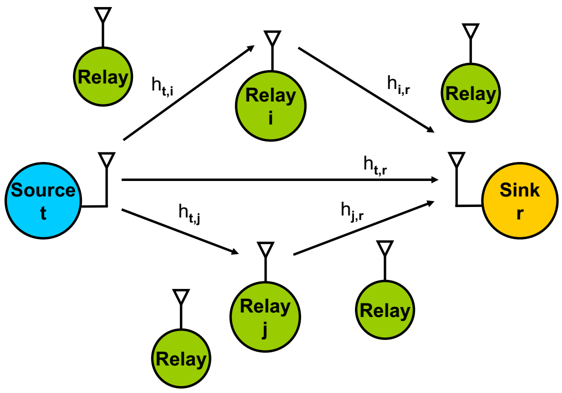

Figure 1: Two-hop wireless network.

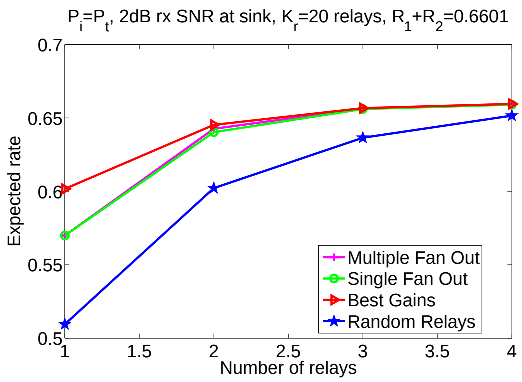

Figure 2: Expected rate as a function of number of selected relays.

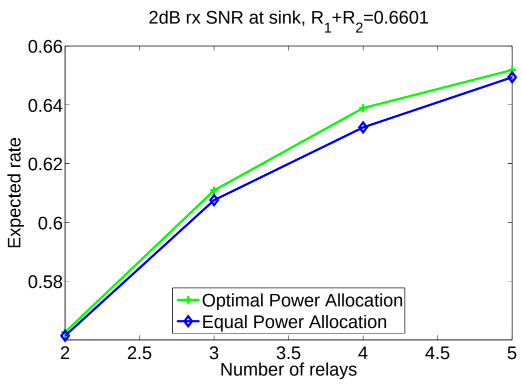

Figure 3: Expected rate for two relay power allocation strategies.

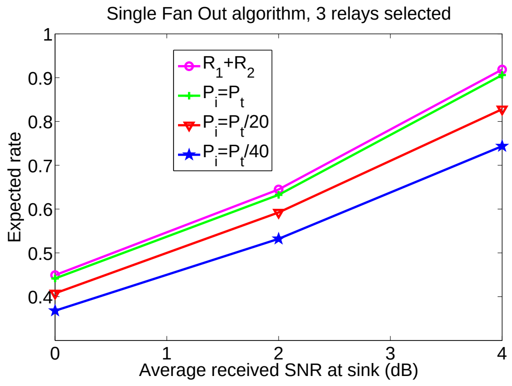

Figure 4: Expected rate as a function of average received SNR at destination.

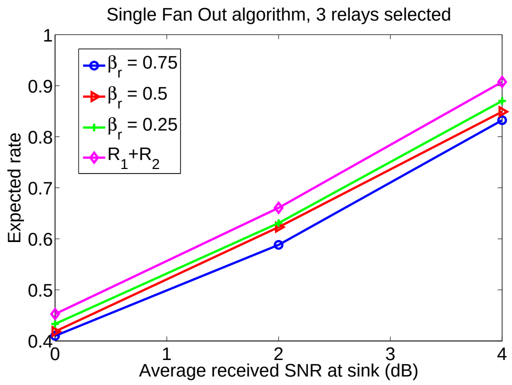

Figure 5: Expected rate as a function of power split at relays.

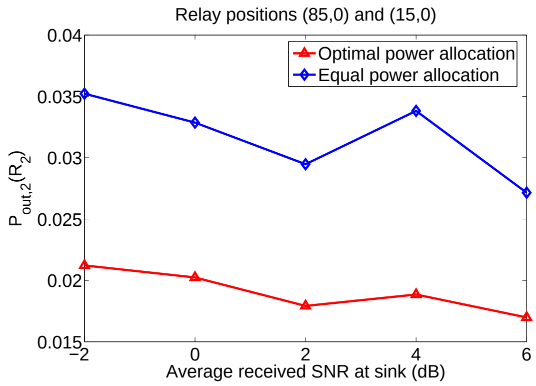

Figure 6: Outage probability for R_2 with 1 good relay and 1 bad relay.

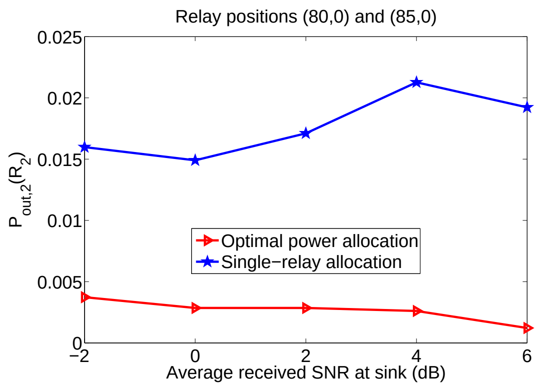

Figure 7: Outage probability for R_2 with 2 good relays.

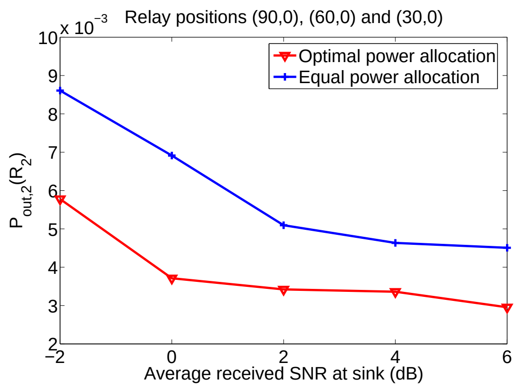

Figure 8: Outage probability for R_2 with 1 good, 1 average and 1 bad relay.