Routing is Order-optimal in Broadcast Erasure Networks with Interference

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***Abstract*— THIS PAPER IS ELIGIBLE FOR THE STUDENT PAPER AWARD. The transport capacity of a class of erasure networks with broadcast and interference constraints is studied in this paper. A memoryless network model is considered, with transmitted symbols constrained to belong to a finite field. Connections between nodes are modeled to be independent erasures, with the probability of the existence of a "link" between any two nodes decaying exponentially with increasing geographic distance between those two nodes.**

Each node obeys a broadcast requirement. In addition, each receiver obtains the finite-field sum of the unerased symbols sent along all the edges connecting to it (an interference condition). In this setting, the transport capacity is bounded above by a linear growth term in the number of nodes, for any network which obeys a minimum node separation constraint. Finally, we show that this linear growth is achievable in random networks by employing routing.

The main thrust of this paper is its conclusion: Routing is order-optimal in a random broadcast erasure network. Thus, network coding can only provide a constant gain in performance.

I. Introduction

There is a growing interest in the study of the capacity of erasure networks with constraints that reflect the underlying physical layer [1]. One of the primary techniques used to study such networks is network coding. Network coding was first used to achieve the multicast capacity of deterministic wireline networks [2], and was subsequently adopted to study deterministic networks with broadcast constraints [3], and multiple-access constraints [4], [5]. In conjunction with this effort, wireless erasure networks with broadcast constraints (no interference constraints) were studied in [1] with independent erasures between nodes using both random coding and random linear encoding techniques.

Interestingly, a significant emphasis of the effort in this direction has been on studying unicast or multicast transmission. There is great interest in multiple unicasts [6] and multi-source multi-destination networks, but these problems are typically particularly difficult to solve. The presence of interference and broadcast constraints complicates this problem even further.

To get a meaningful handle on understanding more general configurations of erasure networks, this paper takes the approach of determining the transport capacity of such networks. Introduced by Gupta and Kumar [7], transport capacity is defined to be the distance-weighted sum of rates of a network, and provides a convenient scalar description of the amount of traffic that a network can support. For a wireless network with additive Gaussian noise, Xie and Kumar [8] provided a information-theoretic scaling law that shows, under certain high-attenuation conditions, the transport capacity can grow no faster than linearly in the number of nodes in the network. Franceschetti et al. [9] demonstrate that linear growth in the number of nodes is achievable in a random network with an additive Gaussian noise model using routing alone (and no network coding).

Our goal in this paper is to show that routing is order-optimal (in terms of the transport capacity) in a *random* erasure network that has, in its model, both broadcast and interference conditions built-in. Our links are modeled as possessing independent erasures, where the erasure probability depends on the distance between corresponding transmitter-receiver pair. This is used to model the network-layer perspective of wireless networks, where a packet that cannot be successfully decoded is registered as an "erasure", and the probability of successful decoding decays with distance.[1] The interference is modeled as the finite-field sum of received symbols.

Our proof technique can be outline as follows: We provide an information-theoretic outerbound for the transport capacity, finding it to be linear in the number of nodes. Next, we show that, for a random network, this linear growth in capacity is achievable using routing arguments. Thus, network coding can at most provide a constant gain in performance for random broadcast erasure networks with an arbitrary number of source-destination pairs.

The paper begins with a discussion of our system model, the broadcast erasure network with additive interference, in Section II. Our main result consists of two components. In Section III we show that for an arbitrary distribution of sources, destinations, and node placement, no matter how "clever" a master planner may be, the distance-weighted sum of feasible rates grows no faster than linearly in the number of nodes. In Section IV, we show that linear growth is achievable in a random planar network whose area grows linearly, thus

[1] In [10], we applied a similar model to the wireless erasure network model introduced in [1].

maintaining a constant node density.

II. Network Model

Consider a network of $n+1$ nodes, each with a transmitter and a receiver. These nodes are placed on the plane with the sole constraint that each node is separated from its neighbors by a distance of at least d_{min}. The nodes are indexed by the variable $j = 1...n+1$.

The network contains L source-destination nodes pairs (s_l, t_l). We desire to reliably decode each of the L independent information sources, each available to one of the source nodes s_l, at the corresponding destination node t_l at the rate R_l.

Let $d(l)$ be the distance between the source node s_l and destination node t_l. The transport capacity of a network is the supremum of the distance-weighted sum of reliable rates

$$T = \sup_{feasible\{R_1,...,R_L\}} \sum_{l=1}^{L} R_l d(l) \tag{1}$$

where the supremum is taken over all possible sets of source-destination pairs.

Because of the broadcasting requirement of the wireless medium, we can consider the network to be a complete graph: in each timeslot t, each node i chooses a single symbol $X_i(t)$ from the finite-field F_q to broadcast to all other nodes. Our model is an additive-interference erasure network model, similar to that studied in [1] (with the addition of interference) and [5]. Specifically, this means that in each timeslot, the node j receives the sum $Y_j(t)$, where

$$Y_j(t) = \gamma_j(t) \sum_{i \neq j, i=1}^{n+1} h_{ij}(t) X_i(t). \tag{2}$$

In Equation (2), the $\gamma_j(t)$ and $h_{ij}(t)$ are independent (over both indices and timeslots) zero-one binary random variables that take the value 0 with probability ϵ_j and ϵ_{ij}, respectively. We also assume that after receiving $Y_j(t)$, the j^{th} node knows the channel states $\gamma_j(t)$ and $h_{ij}(t)$, for all i.

The erasure probabilities ϵ_{ij} are modeled as an increasing function of distance. The decay parameter α, where $0 < \alpha < 1$, will determine the critical distance d^* over which successful transmissions are likely. Let the erasure probability between two nodes i and j located distance d_{ij} apart from each other be

$$\epsilon_{ij} = 1 - \alpha^{d_{ij}} = 1 - e^{-d_{ij}/d^*}. \tag{3}$$

The event that the entire received sum at the j^{th} receiver is erased has a given constant probability ϵ_j. We assume that there is some maximum value of ϵ_j which is independent of n.

III. Upper Bound

In this section, we show an upper bound for the transport capacity of a broadcast erasure network with additive interference. Before the main proof, we will prove and evaluate a cut-set bound for the sum-rate of reliable information flow across a partition of the nodes.

In the following, we will assume that ϵ_j is zero, that is, there are no erasures of the complete received message at any node. Because we can only lose information with complete erasures, the upper bound obtained with this change remains valid. Since we assume that the receiver j knows not only the symbol, but also the channel state for all the channels which terminate at node j, define $Y_j^*(t)$ to be the vector consisting of both $Y_j(t)$ and all channel states $h_{ij}(t)$, for $i \in 1..n+1$.

Assemble the random variables into vector notation as follows: For any subset S of nodes, consider $X_S(t)$, $X_{S^C}(t)$, $Y_{S^C}(t)$ which are defined intuitively. Define matrices $H_1(t)$ and $H_2(t)$ such that

$$Y_{S^C}(t) = H_1(t) X_S(t) + H_2(t) X_{S^C}(t). \tag{4}$$

Then $Y^*(t)$ is the collection $(Y(t), H_1(t), H_2(t))$.

Define $V(t)$ as the received vector $Y_{S^C}(t)$, under the situation that the nodes in S^C did not have any transmitters:

$$V(t) = H_1(t) X_S(t) \tag{5}$$

Also, define $V^*(t)$ and the collection of $(V(t), H_1(t))$.

Define X^T as the combined vector of all $X(t)$ vectors, $t \in 1..T$. Similarly define X_S^T, $X_{S^C}^T$, V^T, V^{*T}, $Y_{S^C}^T$, and $Y_{S^C}^{*T}$.

Over T timeslots, we wish to transmit messages from some set of L source-destination node pairs at rates $R(l)$, $l \in 1..L$. The l^{th} source-destination pair has source node $s(l)$ and destination node $d(l)$. Let W_{cut} be the vector of messages whose source nodes are in S and whose destination nodes are in S^C. W_{S^C} will be the messages whose source nodes are in S^C, regardless of their destinations. All messages $W(l)$ are independent of each other and uniformly chosen from the set $(1, 2, .., 2^{TR(l)})$.

Lemma 1: If a set of rates $R(l)$ are achievable, then

$$\sum_{s(l) \in S, d(l) \in S^C} R(l) \leq I(X_S; V | H_1) \tag{6}$$

for some joint distribution $p(x_1, x_2, .., x_{n+1})$. Further, this cut-set bound evaluates to the expected value of the rank (in F_q) of the random matrix H_1 multiplied by $\lg q$.

Proof: The proof is based on the work of Xie and Kumar [8] and the text by Cover and Thomas.[11]

Starting from the fact that the rate across the cut is equal to the entropy of the messages that go across the cut:

$$T \sum_{s(l) \in S, d(l) \in S^C} R(l) = H(W_{cut}) \tag{7}$$

$$= I(W_{cut}; V^T, H_1^T, H_2^T, W_{S^C}) + H(W_{cut} | V^T, H_1^T, H_2^T, W_{S^C}) \tag{8}$$

$$\leq I(W_{cut}; V^T, H_1^T, H_2^T, W_{S^C}) + T\epsilon_T \tag{9}$$

where Equation (9) comes from Fano's inequality and the fact that

$$W_{cut} \to (V^T, H_1^T, H_2^T, W_{S^C}) \to (Y_{S^C}^{*T}, W_{S^C}) \tag{10}$$

forms a Markov chain. The messages W_{cut} are decoded from knowledge of Y_{S^C}, W_{S^C}, H_1^T, and H_2^T.

Returning to Equation (9), the steps continue

$$= I(W_{cut}; W_{S^C}) + I(W_{cut}; V^{*T}, H_2^T | W_{S^C}) + T\epsilon_T \quad (11)$$

$$= 0 + H(V^{*T}|H_2^T, W_{S^C}) + H(H_2^T|W_{S^C}) - H(H_2^T|W_{S^C}, W_{cut}) - H(V^{*T}|H_2^T, W_{cut}, W_{S^C}) + T\epsilon_T \quad (12)$$

$$= H(V^{*T}|H_2^T, W_{S^C}) - H(V^{*T}|H_2^T, W_{cut}, W_{S^C}) + T\epsilon_T \quad (13)$$

$$\leq H(V^{*T}) - H(V^{*T}|H_2^T, W_{cut}, W_{S^C}) + T\epsilon_T \quad (14)$$

Equations (12 and 13) follow because messages W_l and channel states h_{ij} are independent.

We'll now examine the second term in Equation (14).

$$\begin{aligned} &H(V^{*T}|H_2^T, W_{cut}, W_{S^C}) \\ &= \sum_{t=1}^{T} H(V^*(t)|V^{*t-1}, W_{cut}, W_{S^C}, H_2^T) \quad (15) \\ &\geq \sum_{t=1}^{T} H(V^*(t)|X_S(t), W_{cut}, W_{S^C}, H_2^T) \quad (16) \\ &= \sum_{t=1}^{T} H(V^*(t)|X_S(t)) \quad (17) \end{aligned}$$

Equation (17) follows because

$$(V^{*t-1}, W_{S^C}, W_{cut}, H_2^T) \to X_S(t) \to V^*(t) \quad (18)$$

is a Markov chain, since $V(t) = H_1(t)X_S(t)$.

Thus,

$$\begin{aligned} T \sum_{s(l)\in S, d(l)\in S^C} R(l) &= H(W_{cut}) \\ &\leq \sum_{t=1}^{T} I(X_S(t); V^*(t)) + T\epsilon_T \\ &= \sum_{t=1}^{T} I(X_S(t); V(t)|H_1(t)) + T\epsilon_T \quad (19) \end{aligned}$$

since X_S and H_1 are independent, which shows the first part of the lemma. Now, examine each mutual information term $I(X_S(t); V(t)|H_1(t))$ which equals $H(V(t)|H_1(t))$ since $V(t)$ is a deterministic function of $H_1(t)$ and $X_S(t)$.

Maximizing the entropy in the vector V is achieved by making all of the elements in $X_S(t)$ independent random variables uniformly distributed over the field F_q and observing that

$$\begin{aligned} &H(V(t)|H_1(t)) \\ &= H(V_1(t)|H_1(t)) + H(V_2(t)|H_1(t), V_1(t)) + ... \\ &\quad + H(V_m(t)|V^{m-1}(t), H_1(t)) \quad (20) \end{aligned}$$

where $m = |S_C|$, $V_k(t)$ is the received symbol at the k^{th} node in S_C, and $V^k(t)$ is the collected vector $(V_1(t), ..., V_k(t))$. Each term in Equation (20) has a maximum value of $\lg q$, and, given a particular instance of the transfer matrix $H_1(t)$, if a term can be written as a linear combination of terms with smaller indices, then the conditional entropy of that term is zero. Thus, the value of $H(V(t)|H_1(t))$ is $\lg q$ times the number of linearly independent elements of the vector $V(t)$, or in other words, the rank of the matrix $H_1(t)$ in the finite field F_q times $\lg q$. Taken over all possible instances of the matrix $H_1(t)$, the cutset bound on rates across the cut becomes the expected value of the rank of $H_1(t)$ times $\lg q$. ■

Now follows the main theorem describing the upper bound.

Theorem 1: The transport capacity for any broadcast erasure network with additive interference with $n+1$ nodes subject to a d_{min} minimum separation constraint and decay parameter α is less than Cn, where C is a constant depending only on α and d_{min}.

Proof: Lemma 1 gives us a cut-set upperbound on the total rate of information flow across a partition of the network. For a network of $n+1$ nodes, we will define and examine the rate bounds across $4n$ partitions, defined as follows.

First, we will assign a label (i_1, i_2) to each node. The distinct indexes i_1, where $1 \leq i_1 \leq n+1$ are assigned in order of increasing x-coordinate. When two or more nodes share the same x-coordinate, arbitrarily assign the order of those indexes. Similarly, assign each node an index i_2 in order of increasing y-coordinate.

Define S_m^h to be the set of nodes with labels such that $i_1 \leq m$, where $1 \leq m \leq n$. Similarly define S_m^v to be the set of nodes with labels such that $i_2 \leq m$, for all $1 \leq m \leq n$. We can now examine four rate bounds:

- Let R_m^h be the cut-set bound on the sum of rates from the sources in S_m^h to the destinations in ${S_m^h}^C$.
- Let ${R_m^h}'$ be the cut-set bound on sum of the rates from the sources in ${S_m^h}^C$ to the destinations in S_m^h.
- Similarly define R_m^v and ${R_m^v}'$ as bounds across the vertical partitions.

Finally, let d_m^h be the horizontal projection of the distance between the two nodes whose index i_1 are m and $m+1$, and d_m^v be the vertical projection of the distance between the nodes with index i_2 of m and $m+1$. The partitions S_m^h now represent *vertical* cuts and the R_m^h are bounds on the rate flow in the horizontal direction.

Using this notation, we can bound the total transport capacity of our network as

$$\begin{aligned} T &= \sup_{feasible\{R_1,...,R_L\}} \sum_{l=1}^{L} R_l d(l) \quad (21) \\ &\leq \sum_{m=1}^{n} d_m^v \left(R_m^v + {R_m^v}'\right) + \sum_{m=1}^{n} d_m^h \left(R_m^h + {R_m^h}'\right). \quad (22) \end{aligned}$$

We will only show the derivation of the bound for one of the four sets of cuts, since the analysis of the other three sets will be identical. Let the portion of the transport capacity bound corresponding to the information flow across horizontal cuts (data flow in the positive vertical direction) be

$$T_v = \sum_{m=1}^{n} d_m^v R_m^v \tag{23}$$

$$= \sum_{m=1}^{n} d_m^v E\left[rank\left(H_{1m}^v\right)\right] \tag{24}$$

where H_{1m}^v is the random matrix describing the erasures which take place between transmitters and receivers on opposite sides of the partition S_m^v, specifically

$$Y_{S_m^v{}^C}(t) = H_{1m}^v(t) X_{S_m^v}(t) + H_{2m}^v(t) X_{S_m^v{}^C}(t). \tag{25}$$

Precisely computing the expected value of the rank of a random matrix is a difficult problem [12]. However, to linearly bound the transport capacity of the broadcast erasure network with interference, only a simple bound is required:

Lemma 2: The expected value of the rank of any random matrix is upperbounded by the expected value of the number of non-zero entries in that random matrix.

The proof is obvious.

The entry $[H_{1m}^v]_{j,k-m}(t)$, for $1 \leq j \leq m$ and $m+1 \leq k \leq n+1$ is a Bernoulli random variable which tells us whether the transmission from the node with label $i_1 = j$ was erased or received successfully at the receiver of the node with label $i_1 = k$ at time t. Thus, if we define d_{jk} to be the distance between these two nodes, the entry is non-zero with probability $\alpha^{d_{jk}}$. The expected value of the number of non-zero entries in H_{1m}^v is therefore used to bound

$$R_m^v = (\lg q) E\left[rank(H_{1m}^v)\right] \tag{26}$$

$$\leq (\lg q) E\left[\sum_{j=1}^{m} \sum_{k=m+1}^{n+1} 1_{\left[H_{1m}^v\right]_{j,k-m} \neq 0}\right] \tag{27}$$

$$= \lg q \sum_{j=1}^{m} \sum_{k=m+1}^{n+1} \alpha^{d_{jk}}. \tag{28}$$

Thus,

$$T_v \leq \lg q \sum_{m=1}^{n} d_m^v \sum_{j=1}^{m} \sum_{k=m+1}^{n+1} \alpha^{d_{jk}} \tag{29}$$

Now, let d_{jml} be the distance from the j^{th} node to the l^{th} closest node across the m_{th} cut. (Note that this is not necessarily that same as the distance between the j^{th} and $(m+l)^{th}$ nodes.)

$$d_{jml} \geq \sqrt{\left(\sum_{i=j}^{m} d_i^v\right)^2 + \frac{d_{min}^2}{3}(l)} - \frac{d_{min}}{2} \tag{30}$$

The bound comes from packing l circles of radius $d_{min}/2$ into a semi-circle whose near side is at least $\sum_{i=j}^{m} d_i^v - d_{min}/2$ away from node j.

Thus, Equation (29) is upperbounded as

$$T_v \leq (\lg q) \alpha^{-d_{min}/2} \sum_{m=1}^{n} \sum_{j=1}^{m} d_m^v \sum_{k=1}^{\infty} \alpha^{\sqrt{\left(\sum_{i=j}^{m} d_i^v\right)^2 + k d_{min}^2/2}} \tag{31}$$

and it can be shown that $T_v \leq Cn$ for some constant C depending only on d_{min} and d^* (see, for example, Theorem 3 in [10]). ■

Thus, no matter how one positions nodes and assigns sources, destinations, and rate-pairs, the total transport capacity of a broadcast erasure network with additive finite-field interference grows no faster than linearly in the number of nodes n.

IV. Achievability in Random Networks

Our proof of the achievability of $\Omega(n)$ transport capacity follows from the work of M. Franceschetti et al [9]. We shall summarize their prior results of which we will make use and be more precise with the exposition of those changes that are particular to our model.

The authors of [9] examine a Gaussian interference network of size $\sqrt{n} \times \sqrt{n}$, and show that for a random matching of source-destination pairs on a uniformly randomly distributed set of nodes, a per-node throughput capacity of $\Omega(1/\sqrt{n})$ bit/sec is achievable. The main idea of the proof is to divide the network into horizontal rectangles and show that there exit "highways," or disjoint paths which can be used to carry data from the left edge to the right edge of the network. Similarly, a known density of vertical highways also exist, and this mesh of paths transports the information from sources to destinations. The network operation protocol has three parts: A draining phase, where data is routed from source nodes to the highways, a transport phase, where data travels along the highways, and a distribution phase, where the data is routed from the highway to its final destination node.

Theorem 2: A random broadcast erasure network with interference can achieve a per-node throughput capacity of $\Omega(1/\sqrt{n})$ bit/sec, and hence a total transport capacity of $\Omega(n)$.

The construction of the highways and the routing protocol are identical to the procedure in [9]. Our contribution is determine necessary conditions for the required rate to be achievable in our erasure model, and to show that those conditions are met.

Nodes are distributed according to a Poisson process of unit intensity on square of area n. The total area is divided into smaller squares of side-length c, and a horizontal "highway" consists of a collection these smaller squares, each containing at least one node, which form a continuous path across the total area.

Our proof proceeds as follows: First, we will show the analog of Theorem 3 in [9], namely that there exists a rate $R(d) > 0$ such that a node in our network can transmit w.h.p. at rate $R(d)$ to any destination within a distance d using a TDMA scheme on squares of side length c. Further, as d tends to infinity,

$$R(d) = \Omega\left(d^{-2} e^{-d/d^*}\right). \tag{32}$$

Next, the network is divided in horizontal slabs of constant width $c\kappa \ln \sqrt{n/c^2}$, where the parameter κ is to be later determined, and consecutively assign highways to slabs. From [9], Theorem 5 we see that w.h.p., each node is at most a distance $d \leq \sqrt{2}c\kappa \ln \frac{\sqrt{n}}{\sqrt{2}c}$ from its highway. Substituting this d into $R(d)$ in Equation 32 and appropriately choosing κ, we find the rate that one node in a square of side length c can transmit data to the highway. There may be as many as $\ln \frac{\sqrt{n}}{c}$ nodes in a square which will have to share this rate. Transmitting data from the highway back to a destination node (the distribution phase) is the dual problem, and therefore is possible to perform at the same rate.

In the highway phase, we note that each node is no more than a distance $2\sqrt{2}c$ from the next node along the highway, so each highway can carry data at a constant rate. There are no more than $O(\sqrt{n})$ nodes in any slab, so the highway can devote a $1/\sqrt{n}$ fraction of its (constant) throughput to each node. As long as the $R(d)$ achievable in the draining and distribution phases is $\Omega(1/\sqrt{n})$, then a throughput of rate $\Omega(1/\sqrt{n})$ is achievable. In summary, there are $n/2$ random transmit-receive pairs, each providing a rate $\Omega(1/\sqrt{n})$ over a distance $\Omega(\sqrt{n})$, for a transport capacity of $\Omega(n)$.

Proof: Assume that nodes in the network wish to transmit data to destination nodes at most a distance d away. We will say that the transmission was successful only if both 1) the symbol sent by the transmitting node was not erased at the receiver and 2) any symbols sent by simultaneously transmitting nodes are all erased. We will operate the network in a TDMA scheme with $T = (kd/c)^2$ timeslots, where k is to be determined.

If the intended receiver is at most a distance d from the transmitter, then it is at most d/c squares away from the transmitter. Under the TDMA scheme, the nearest simultaneously operating transmitter is at least $(k-1)\frac{d}{c} - 1$ squares away, and the 8 closest transmitters are all (at least) this distance from the receiver. The next 16 operating transmitters are all at least $(2k-1)\frac{d}{c} - 1$ squares away, and so on so that there are $8i$ transmitters at least distance of $(ik-1)d - c$ from the intended receiver, for all positive integers i.

The union bound on the probability that the symbol from at least one of these transmitter is not erased, P_{int}, is

$$P_{int} \leq \sum_{i=1}^{\infty} 8ie^{-((ik-1)d-c)/d^*}$$
$$= 8e^{(d+c)/d^*} \frac{e^{-kd/d^*}}{\left(1 - e^{-kd/d^*}\right)^2}. \tag{33}$$

We can show that by choosing

$$k > 1 + \left(d^* \ln 32 + c\right)/d \tag{34}$$

so that the TDMA scheme operates in

$$T = \lceil (d/c + d^* \ln 32/c + 1)^2 \rceil \tag{35}$$

timeslots, $P_{int} < 1$ and thus

$$R(d) \geq e^{-d/d^*}(1 - P_{int})T^{-1}. \tag{36}$$

Using that facts that in the draining and distribution phases, $d \leq \sqrt{2}c\kappa \ln \frac{\sqrt{n}}{\sqrt{2}c}$, that the number of timeslots $T = \Theta(d^2)$, and that there may be $O(\ln n)$ nodes in each square, we see that the achievable rate in these phases is

$$\Omega\left(e^{-d/d^*}(1 - P_{int})d^{-2}(\ln n)^{-1}\right) \tag{37}$$
$$= \Omega\left(e^{-\sqrt{2}c\kappa \ln \frac{\sqrt{n}}{\sqrt{2}c}/d^*} (\ln n)^{-3}\right) \tag{38}$$
$$= \Omega\left(n^{-\sqrt{2}c\kappa/2d^*} (\ln n)^{-3}\right) \tag{39}$$

As long as we choose

$$\sqrt{2}c\kappa/2d^* < 1/2 \tag{40}$$

while keeping

$$c^2 > \ln 6 + 2/\kappa \tag{41}$$

to fulfill the requirements of [9], Theorem 5, a per-node throughput of $\Omega(1/\sqrt{n})$ in our random broadcast erasure network with interference is achievable. ∎

V. Conclusion

We have shown that the transport capacity of a broadcast erasure network with interference can grow no faster than linearly in the number of nodes. Further, we have shown that in the random extended network case, linear growth is achievable using a simple routing strategy. We conclude that, for random multiple-source multiple-destination broadcast erasure networks with interference, network coding provides no order-wise improvement in transport capacity. Future work includes showing similar bounds for a model where the erasure probability decays polynomially, instead of exponentially, with increasing internode distance.

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